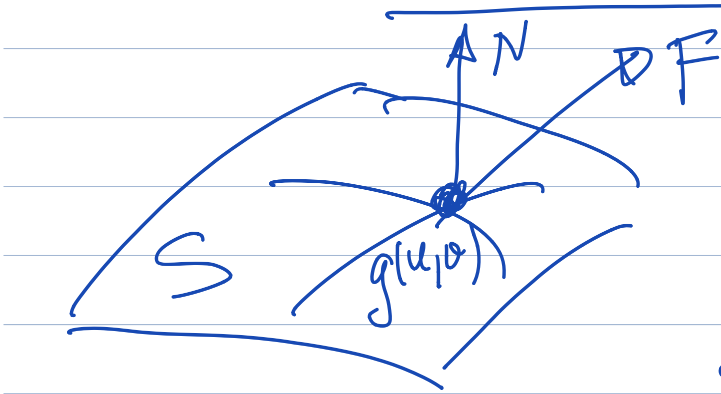
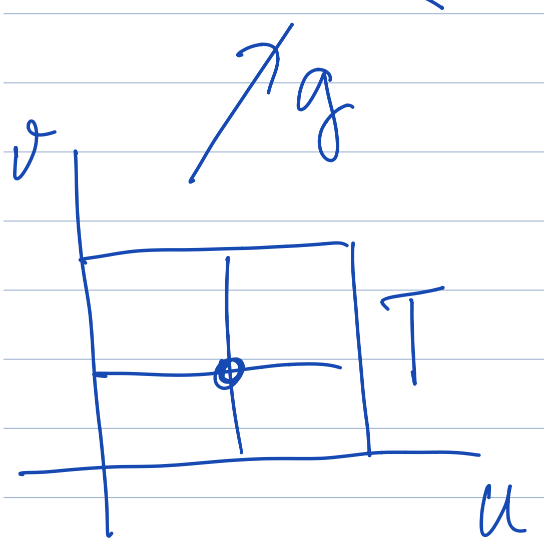
CDI-II — Prática 1/6/21

Ficha 12, Ficha 13

F.12 — Fluxo, T. divergência.



$$\int_S F\cdot N \equiv \iint_S F\cdot N =$$

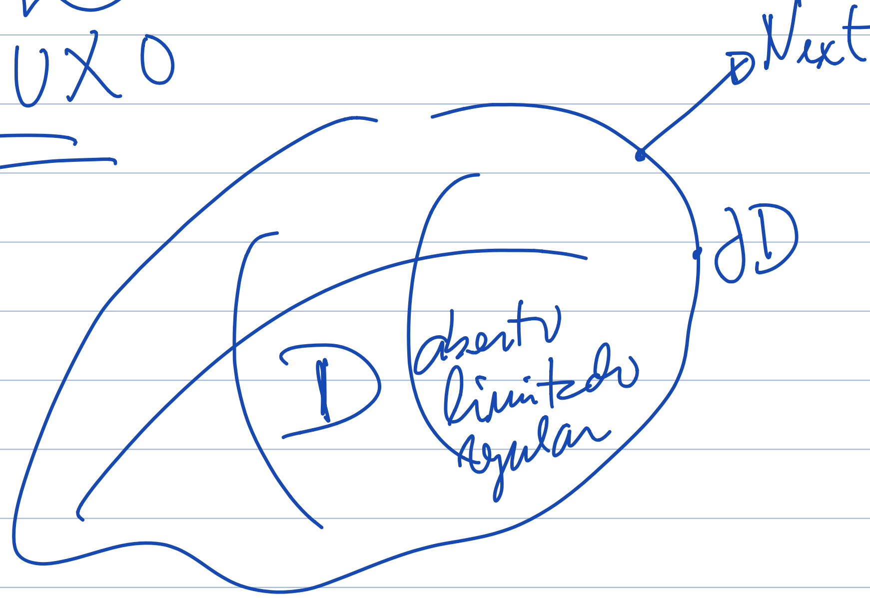


$$= \pm \iint_T F(g(u,v))\cdot \underbrace{D_u g \times D_v g}\, du\, dv$$

T. divergência:

$$\iint_{\partial D} F \cdot N_{ext} = \iiint_{D} \operatorname{div} F$$

FLUXO



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$\boxed{F.12}$

4 — Dados: H, A, n

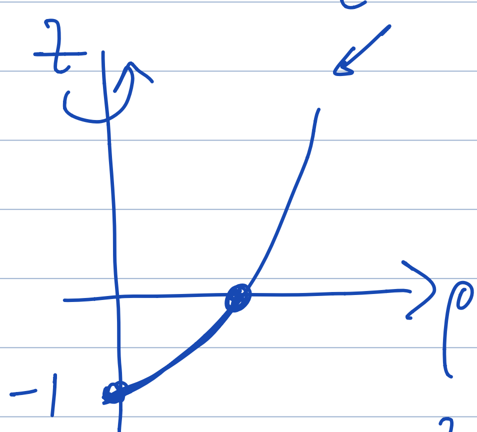
$$\int_A H \cdot n = ? \qquad \boxed{n_z < 0}$$

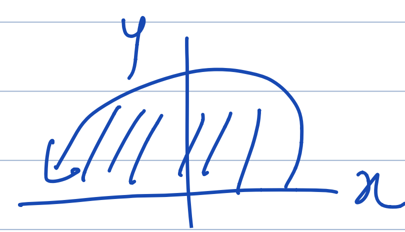
a) Parametrizar A:

$$z = x^2 + y^2 - 1 \ , \ z < 0 \ , \ y > 0$$

(ρ, θ, z):

$$z = \rho^2 - 1 \ , \ z < 0 \ , \ y > 0$$





$$0 < \theta < \pi$$

$$\rho^2 - 1 < 0 \iff \rho < 1$$

$$g(\rho,\theta) = (\rho\cos\theta, \rho\,\text{sen}\,\theta, \rho^2 - 1)$$

$$T: \quad 0 < \rho < 1$$
$$\quad\;\; 0 < \theta < \pi$$

2)

$$\begin{array}{c} \quad\; i \qquad\qquad j \qquad\qquad k \\ \begin{vmatrix} D_\rho g = (\cos\theta, \text{sen}\,\theta, 2\rho) \\ D_\theta g = (-\rho\,\text{sen}\,\theta, \rho\cos\theta, 0) \end{vmatrix} \end{array} \Big\} \text{tangentes}$$

$$-D_\rho g \times D_\theta g = (+2\rho^2\cos\theta, +2\rho^2\,\text{sen}\,\theta, -\boxed{\rho}) \quad \text{Normal!}$$

$$\rho > 0 \qquad \boxed{N_z < 0}$$

$$H(g(\rho,\theta)) = (-\rho\,\text{sen}\,\theta, \rho\cos\theta, \rho^2 - 1)$$

$$-H(g(\rho,\theta)) \cdot D_\rho g \times D_\theta g = -\rho(\rho^2 - 1)$$

3) $\displaystyle\iint_A H \cdot n = -\int_0^{\pi} \left(\int_0^{1} \rho(\rho^2 - 1)\, d\rho \right) d\theta$

etc...

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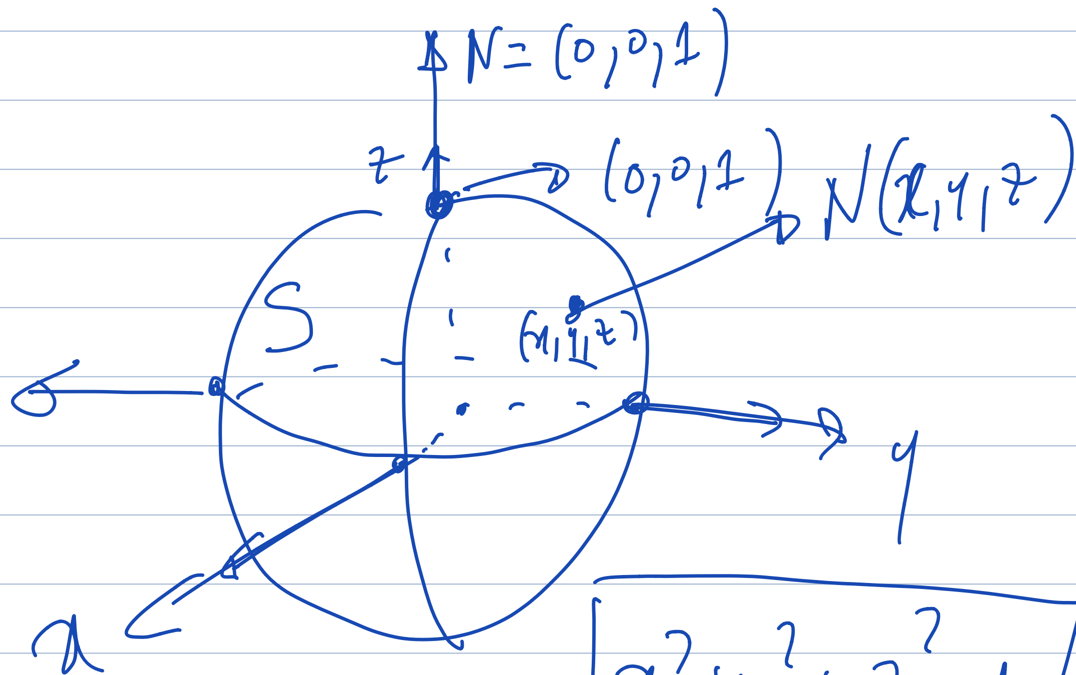
6 — $F(x, y, z) = h(r)(x, y, z)$

$r = \|(x, y, z)\| = \sqrt{x^2 + y^2 + z^2}$

$h :]0, +\infty[\longrightarrow \mathbb{R}$

$S : \quad x^2 + y^2 + z^2 = 1$

$N : \quad N(0, 0, 1) = (0, 0, 1)$



$$\boxed{x^2+y^2+z^2=1} \quad \boxed{R=1}$$

$$N(x,y,z) = \frac{(2x,2y,2z)}{\sqrt{4x^2+4y^2+4z^2}} = (x,y,z)$$

$$F(x,y,z) = h(R)(x,y,z)$$

em S : $F(x,y,z) = h(1)(x,y,z)$

$\boxed{R=1}$

em S: $F \cdot N = h(1)\underbrace{(x^2+y^2+z^2)}_{=1} = h(1)$

$$F \cdot N = h(1) \quad \text{en } S$$

(constante)

$$\int_S F \cdot N = \int_S h(1) = h(1) \int_S 1 = h(1)\, \mathrm{vol}_2(S)$$

$$= 4\pi\, h(1).$$

8 - $D \subset \mathbb{R}^3$, $\quad vol_3(D) = ?$

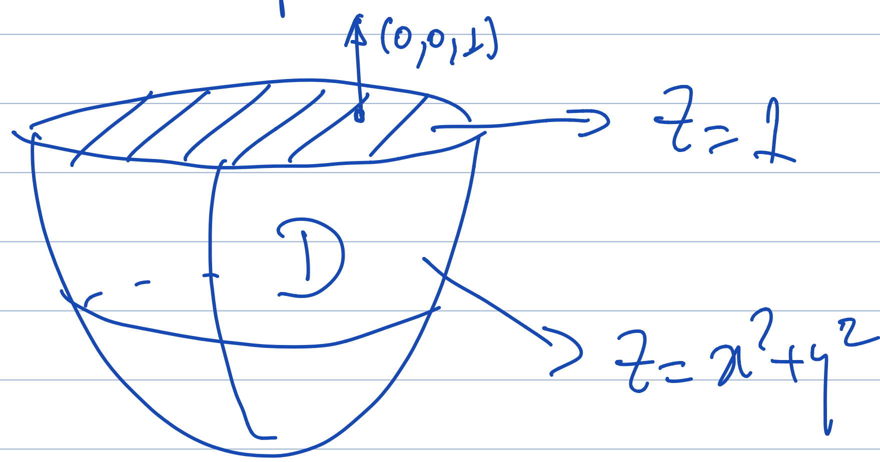
$$\underbrace{\iiint_D \operatorname{div} F}_{?} = \iint_{\partial D} F \cdot N_{ext}$$

calcular

Se $\underbrace{\operatorname{div} F = 1}$ então $\iiint_D \operatorname{div} F = vol_3(D)$

- $\boxed{F(x,y,z) = (x,0,0)}$
- $F(x,y,z) = (0,y,0)$
- $F(x,y,z) = (0,0,z)$

$D: \quad x^2+y^2 < z < 1$



$F(x,y,z) = (x,0,0)$, $\operatorname{div} F = 1$

En $z=1$; $x^2+y^2<1$, $N=(0,0,1)$

$F=(x,0,0)$

$F \cdot N = 0$

En $z=x^2+y^2$; $z<1$:

parametrizar: $z=\rho^2$, $\rho<1$

$g(\rho,\theta) = (\rho\cos\theta, \rho \operatorname{sen}\theta, \rho^2)$ etc... $0<\theta<2\pi$

9 – Dados: S, F, $N_z \geq 0$

$$\iint_S F \cdot N = ?$$

$$\iint_{\partial D} F \cdot N_{ext} = \iiint_D \operatorname{div} F$$

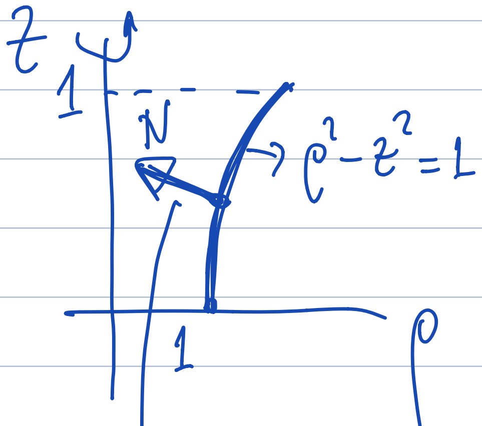
Solução: 1) Construir D tal que $S \subseteq \partial D$.

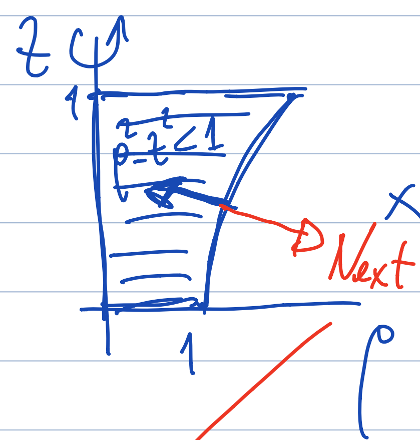
de S

$S \longrightarrow D$ $\dfrac{\text{aberto}}{\dfrac{\text{limitado}}{\text{Regular}}}$

$\boxed{=}$ 1 eq. $\qquad\qquad$ $\boxed{\text{Inequações}}$

$$\begin{cases} x^2+y^2=1+z^2, \\ 0<z<1 \end{cases} \longrightarrow \begin{cases} x^2+y^2<1+z^2 \\ 0<z<1 \end{cases}$$

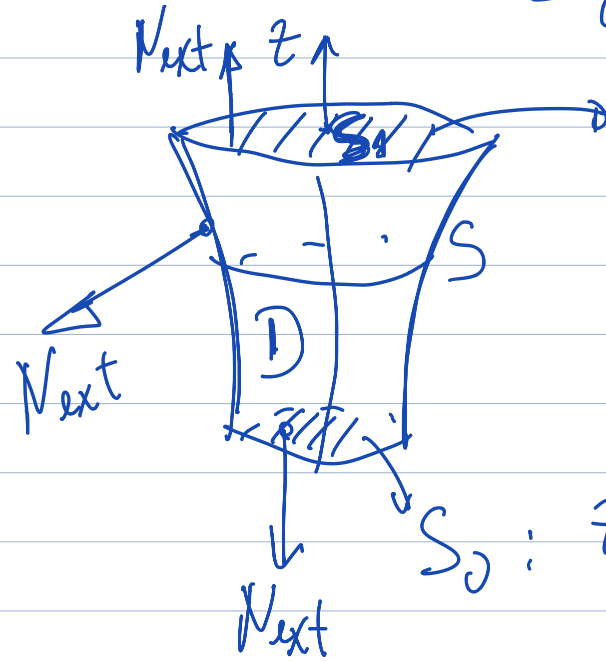




$$N \equiv -N_{ext}$$

$$\partial D = S \cup \underbrace{\begin{cases} z=0 \\ x^2+y^2<1 \end{cases}}_{S_0} \cup \underbrace{\begin{cases} z=1 \\ x^2+y^2<2 \end{cases}}_{S_1}$$

$$\partial D = S \cup S_0 \cup S_1$$



S_1: $z=1$; $x^2+y^2<2$

$N_{ext} = (0,0,1)$

S_0 : $z=0$; $x^2+y^2<1$

$N_{ext} = (0,0,-1)$

$$\boxed{\iint_S F\cdot N_{ext}} + \iint_{S_0} F\cdot N_{ext} + \iint_{S_1} F\cdot N_{ext} = \iiint_D \operatorname{div} F$$

?

Calcular!

etc...

<u>Ficha 13</u>: <u>T. de STOKES</u>.

$$\underbrace{\iint_S \text{rot}\, F \cdot N}_{\text{Fluxo}} = \underbrace{\oint_{\partial S} F \cdot dg}_{\text{Trabalho}}$$

2– Dados: G, L, sentido

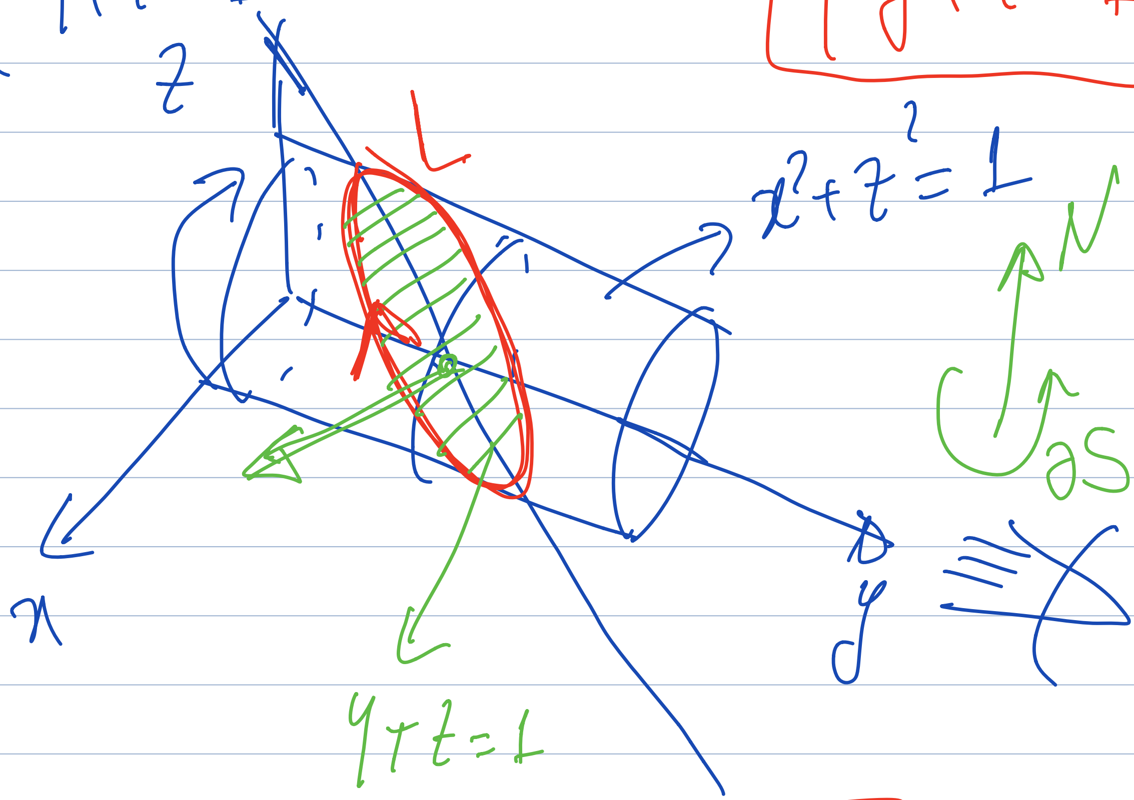
Questão: $\displaystyle\int_L G \cdot dg = \, ?$ (<u>trabalho</u>)

<u>Solução</u>: de L construir S tal que $L = \partial S$.

$L \longrightarrow S$ limitada

$\left.\begin{array}{l} = \\ = \end{array}\right\}$ 2 eq. $\qquad \Leftrightarrow \quad \boxed{1 \text{ eq.}}$

$$\left\{\begin{array}{l} x^2+z^2=1 \\ y+z=1 \end{array}\right. \longrightarrow \boxed{\left\{\begin{array}{l} x^2+z^2 \leq 1 \\ y+z=1 \end{array}\right.}$$



$$\boxed{L = \partial S}$$

etc.